

## CLAIMS

What is claimed is:

1. A method comprising:
  - generating a first covariance matrix from a desired mean vector and a desired covariance matrix of a Bernoulli distribution;
  - constructing a normal vector using the desired mean vector and the first covariance matrix; and
  - generating a sampling vector using the normal vector and a threshold vector, the sampling vector having the desired mean vector and the desired covariance matrix.
2. The method of claim 1 wherein generating the first covariance matrix comprises:
  - generating an integral expression  $F$  for a first non-diagonal element  $s_{ij}$  of the first covariance matrix at a row index  $i$  and a column index  $j$ , the integral expression having an integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$  in the threshold vector at the vector indices  $i$  and  $j$ ; and
  - obtaining the first non-diagonal element  $s_{ij}$  using the integral expression  $F$ , a mean  $\mu_k$  of the desired mean vector, and a desired non-diagonal element  $\Sigma_{ij}$  of the desired covariance matrix.
3. The method of claim 2 further comprising:
  - obtaining a diagonal element  $s_{jj}$  of the first covariance matrix at a first row index  $j$  and a first column index  $j$  using the mean  $\mu_j$  at the vector index  $j$ , the diagonal element being equal to a desired diagonal element  $\Sigma_{jj}$  of the desired covariance matrix.
4. The method of claim 3 further comprising:
  - generating a threshold element  $\tau_j$  of the threshold vector at a vector index  $j$ , the threshold element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1-2\mu_j)$  wherein  $\mu_j$  and  $\sigma_j$  are desired mean and variance, respectively, at the vector index  $j$  and  $\operatorname{inverf}$  is an inverse error function.

1           5.     The method of claim 2 wherein constructing the normal vector comprises:  
2           generating normal elements of the normal vector using the desired mean vector and  
3           the first covariance matrix.

1           6.     The method of claim 5 wherein generating the sampling vector comprises:  
2           comparing a normal element  $Y_k$  of the normal vector at a vector index  $k$  with a  
3           corresponding threshold element  $\tau_k$  of the threshold vector at the vector index  $k$ ;  
4           setting a sampling element of the sampling vector at the vector index  $k$  to a first  
5           value if the normal element  $Y_k$  is greater than the corresponding threshold element  $\tau_k$ ; and  
6           setting the sampling element of the sampling vector at the vector index  $k$  to a  
7           second value if the normal element  $Y_k$  is equal to or less than the corresponding threshold  
8           element  $\tau_k$ .

1           7.     The method of claim 2 wherein generating the integral expression  $F$   
2           comprises:

3           forming a first variable  $\rho = s_{ij}/(\sigma_i\sigma_j)$ ;  
4           forming a second variable  $c = \sqrt{2(1-\rho^2)}$ ;  
5           forming a third variable  $P = (a_i + a_j)/(c\sqrt{2})$ ,  $P$  being one of the integral limits;  
6           forming a fourth variable  $Q = (a_j - a_i)/(c\sqrt{2})$ ; and  
7           forming the integral expression

$$8 \quad F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_P^{\infty} e^{-p^2(1-\rho)} (\operatorname{erf} \sqrt{1+\rho}(Q+p+P) - \operatorname{erf}(\sqrt{1+\rho}(Q-p+P))) dp$$

9           wherein  $p$  is an integral variable,  $\operatorname{erf}$  is an error function,  $a_i$  and  $a_j$  are respectively  
10          equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the vector indices  
11          equal respectively to the row index  $i$  and the column index  $j$ ,  $\mu_i$  and  $\mu_j$  being the means at  
12          the vector indices equal respectively to the row index  $i$  and the column index  $j$ ,  $\sigma_i$  and  $\sigma_j$   
13          being the variances at the vector indices equal respectively to the row index  $i$  and the  
14          column index  $j$ .

- 1           8.       The method of claim 7 wherein obtaining the first non-diagonal element  
2 comprises:  
3           determining a right hand side (RHS) quantity  $g_{ij} = \Sigma_{ij} + \mu_i \mu_j$ ;  
4           equating the integral expression to the RHS quantity to form an integral equation  $F$   
5  $= g_{ij}$ ; and  
6           solving the integral equation for the first variable  $\rho$ .
- 1           9.       The method of claim 8 wherein solving the integral equation comprises:  
2           solving the integral equation using a numerical method.
- 1           10.      The method of claim 6 wherein the first value is 1 and the second value is 0.
- 1           11.      A computer program product comprising:  
2           a machine useable medium having program code embedded therein, the program  
3 code comprising:  
4           computer readable program code to generate a first covariance matrix from  
5 a desired mean vector and a desired covariance matrix of a Bernoulli distribution;  
6           computer readable program code to construct a normal vector using the  
7 desired mean vector and the first covariance matrix; and  
8           computer readable program code to generate a sampling vector using the  
9 normal vector and a threshold vector, the sampling vector having the desired mean  
10 vector and the desired covariance matrix.
- 1           12.      The computer program product of claim 11 wherein the computer readable  
2 program code to generate the first covariance matrix comprises:  
3           computer readable program code to generate an integral expression  $F$  for a first  
4 non-diagonal element  $s_{ij}$  of the first covariance matrix at a row index  $i$  and a column index  
5  $j$ , the integral expression having an integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$   
6 in the threshold vector at the vector indices  $i$  and  $j$ ; and

7 computer readable program code to obtain the first non-diagonal element  $s_{ij}$  using  
 8 the integral expression  $F$ , a mean  $\mu_k$  of the desired mean vector, and a desired non-diagonal  
 9 element  $\Sigma_{ij}$  of the desired covariance matrix.

1 13. The computer program product of claim 12 further comprising:  
 2 computer readable program code to obtain a diagonal element  $s_{jj}$  of the first  
 3 covariance matrix at a first row index  $j$  and a first column index  $j$  using the mean  $\mu_j$  at the  
 4 vector index  $j$ , the diagonal element being equal to a desired diagonal element  $\Sigma_{jj}$  of the  
 5 desired covariance matrix.

1 14. The computer program product of claim 13 further comprising:  
 2 computer readable program code to generate a threshold element  $\tau_j$  of the threshold  
 3 vector at a vector index  $j$ , the threshold element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1 - 2\mu_j)$   
 4 wherein  $\mu_j$  and  $\sigma_j$  are desired mean and variance, respectively, at the vector index  $j$  and  
 5  $\operatorname{inverf}$  is an inverse error function.

1 15. The computer program product of claim 12 wherein the computer readable  
 2 program code to construct the normal vector comprises:  
 3 computer readable program code to generate normal elements of the normal vector  
 4 using the desired mean vector and the first covariance matrix.

1 16. The computer program product of claim 15 wherein the computer readable  
 2 program code to generate the sampling vector comprises:  
 3 computer readable program code to compare a normal element  $Y_k$  of the normal  
 4 vector at a vector index  $k$  with a corresponding threshold element  $\tau_k$  of the threshold vector  
 5 at the vector index  $k$ ;  
 6 computer readable program code to set a sampling element of the sampling vector  
 7 at the vector index  $k$  to a first value if the normal element  $Y_k$  is greater than the  
 8 corresponding threshold element  $\tau_k$ ; and  
 9 computer readable program code to set the sampling element of the sampling vector  
 10 at the vector index  $k$  to a second value if the normal element  $Y_k$  is equal to or less than the  
 11 corresponding threshold element  $\tau_k$ .

17. The computer program product of claim 12 wherein the computer readable program code to generate the integral expression F comprises:

computer readable program code to form a first variable  $\rho = s_{ij}/(\sigma_i\sigma_j)$ ;

computer readable program code to form a second variable  $c = \sqrt{2(1-\rho^2)}$ ;

computer readable program code to form a third variable  $P = (a_i + a_j)/(c\sqrt{2})$ , P being one of the integral limits;

computer readable program code to form a fourth variable  $Q = (a_j - a_i)/(c\sqrt{2})$ ; and

computer readable program code to form the integral expression

$$F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_p^{\infty} e^{-p^2(1-\rho)} (\operatorname{erf} \sqrt{1+\rho}(Q+p+P) - \operatorname{erf}(\sqrt{1+\rho}(Q-p+P))) dp$$

wherein p is an integral variable, erf is an error function,  $a_i$  and  $a_j$  are respectively equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the vector indices equal respectively to the row index i and the column index j,  $\mu_i$  and  $\mu_j$  being the means at the vector indices equal respectively to the row index i and the column index j,  $\sigma_i$  and  $\sigma_j$  being the variances at the vector indices equal respectively to the row index i and the column index j.

18. The computer program product of claim 17 wherein the computer readable program code to obtain the first non-diagonal element comprises:

computer readable program code to determine a right hand side (RHS) quantity  $g_{ij} = \Sigma_{ij} + \mu_i\mu_j$ ;

computer readable program code to equate the integral expression to the RHS quantity to form an integral equation  $F = g_{ij}$ ; and

computer readable program code to solve the integral equation for the first variable  $\rho$ .

19. The computer program product of claim 18 wherein the computer readable program code to solve the integral equation comprises:

computer readable program code to solve the integral equation using a numerical method.

20. The method of claim 16 wherein the first value is 1 and the second value is 0.

21. A simulator comprises:  
 a network modeler to model a network of free-space optical links;  
 a reliability modeler coupled to the network modeler to evaluate a reliability model for the network; and  
 a random sampler coupled to the network modeler and the reliability modeler to generate random samples for a Bernoulli distribution, the random sampler comprising:  
 a covariance generator to generate a first covariance matrix from a desired mean vector and a desired covariance matrix of the Bernoulli distribution,  
 a normal vector generator coupled to the covariance generator to construct a normal vector using the desired mean vector and the first covariance matrix, and  
 a thresholder coupled to the covariance generator and the normal vector generator to generate a sampling vector using the normal vector and a threshold vector, the sampling vector having the desired mean vector and the desired covariance matrix.

22. The simulator of claim 21 wherein the covariance generator comprises:  
 an integral expression generator to generate an integral expression  $F$  for a first non-diagonal element  $s_{ij}$  of the first covariance matrix at a row index  $i$  and a column index  $j$ , the integral expression having an integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$  in the threshold vector at the vector indices  $i$  and  $j$ ; and  
 a non-diagonal element generator coupled to the integral expression generator to obtain the first non-diagonal element  $s_{ij}$  using the integral expression  $F$ , a mean  $\mu_k$  of the desired mean vector, and a desired non-diagonal element  $\Sigma_{ij}$  of the desired covariance matrix.

23. The simulator of claim 22 wherein the random sampler further comprises:  
 a diagonal element generator to obtain a diagonal element  $s_{jj}$  of the first covariance matrix at a first row index  $j$  and a first column index  $j$  using the mean  $\mu_j$  at the vector index

4 j, the diagonal element being equal to a desired diagonal element  $\Sigma_{jj}$  of the desired  
5 covariance matrix.

1 24. The simulator of claim 23 wherein the random sampler further comprises:  
2 a threshold vector calculator coupled to the first normal vector generator to  
3 generate a threshold element  $\tau_j$  of the threshold vector at a vector index j, the threshold  
4 element being equal to  $\mu_j + \sigma_j \sqrt{2} \operatorname{inverf}(1-2\mu_j)$  wherein  $\mu_j$  and  $\sigma_j$  are the desired mean and  
5 variance, respectively, at the vector index j and inverf is an inverse error function.

1 25. The simulator of claim 22 wherein the normal vector generator generates  
2 normal elements of the normal vector using the desired mean vector and the first  
3 covariance matrix.

1 26. The simulator of claim 25 wherein thresholder comprises:  
2 a comparator to compare a normal element  $Y_k$  of the normal vector at a vector  
3 index k with a corresponding threshold element  $\tau_k$  of the threshold vector at the vector  
4 index k; and  
5 a selector coupled to the comparator to set a sampling element of the sampling  
6 vector at the vector index k to a first value if the normal element  $Y_k$  is greater than the  
7 corresponding threshold element  $\tau_k$  and to set the sampling element of the sampling vector  
8 at the vector index k to a second value if the normal element  $Y_k$  is equal to or less than the  
9 corresponding threshold element  $\tau_k$ .

1 27. The simulator of claim 22 wherein the integral expression generator  
2 generates the integral expression

$$3 \quad F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_p^{\infty} e^{-p^2(1-\rho)} (\operatorname{erf} \sqrt{1+\rho}(Q+p+P) - \operatorname{erf}(\sqrt{1+\rho}(Q-p+P))) dp$$

4 wherein:

5  $\rho = s_{ij}/(\sigma_i \sigma_j)$ ,  $c = \sqrt{2(1-\rho^2)}$ ,  $P = (a_i + a_j)/(c\sqrt{2})$ , P being one of the integral limits,

6  $Q = (a_j - a_i)/(c\sqrt{2})$ , p is an integral variable, erf is an error function,  $a_i$  and  $a_j$  are

7 respectively equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the  
 8 vector indices equal respectively to the row index  $i$  and the column index  $j$ ,  $\mu_i$  and  $\mu_j$  being  
 9 the means at the vector indices equal respectively to the row index  $i$  and the column index  
 10  $j$ ,  $\sigma_i$  and  $\sigma_j$  being the variances at the vector indices equal respectively to the row index  $i$   
 11 and the column index  $j$ .

1 28. The simulator of claim 27 wherein the non-diagonal element generator  
 2 comprises:  
 3 a right hand side (RHS) generator to determines a right hand side (RHS) quantity  $g_{ij}$   
 4  $= \Sigma_{ij} + \mu_i \mu_j$ ;  
 5 an equation solver coupled to the integral expression generator and the RHS  
 6 generator to equate the integral expression to the RHS quantity to form an integral  
 7 equation  $F = g_{ij}$ , and to solve the integral equation for the first variable  $\rho$ .

1 29. The simulator of claim 28 wherein the equation solver solves the integral  
 2 equation using a numerical method.

1 30. The simulator of claim 26 wherein the first value is 1 and the second value  
 2 is 0.

1 31. A system comprises:  
 2 a processor; and  
 3 a memory coupled to the processor, the memory having program code, the program  
 4 code when executed by the processor causing the processor to:  
 5 generate a first covariance matrix from a desired mean vector and a desired  
 6 covariance matrix of a Bernoulli distribution,  
 7 construct a normal vector using the desired mean vector and the first  
 8 covariance matrix, and  
 9 generate a sampling vector using the normal vector and a threshold vector,  
 10 the sampling vector having the desired mean vector and the desired covariance  
 11 matrix.



32. The system of claim 31 wherein the program code causing the processor to generate the first covariance matrix causes the processor to:

generate an integral expression  $F$  for a first non-diagonal element  $s_{ij}$  of the first covariance matrix at a row index  $i$  and a column index  $j$ , the integral expression having an integral limit as function of threshold elements  $\tau_i$  and  $\tau_j$  in the threshold vector at the vector indices  $i$  and  $j$ ; and

obtain the first non-diagonal element  $s_{ij}$  using the integral expression  $F$ , a mean  $\mu_k$  of the desired mean vector, and a desired non-diagonal element  $\Sigma_{ij}$  of the desired covariance matrix.

33. The system of claim 32 wherein the program code, when executed, further causing the processor to:

obtain a diagonal element  $s_{jj}$  of the first covariance matrix at a first row index  $j$  and a first column index  $j$  using the mean  $\mu_j$  at the vector index  $j$ , the diagonal element being equal to a desired diagonal element  $\Sigma_{jj}$  of the desired covariance matrix.

34. The system of claim 33 wherein the program code, when executed, further causing the processor to:

generate a threshold element  $\tau_j$  of the threshold vector at a vector index  $j$ , the threshold element being equal to  $\mu_j + \sigma_j \sqrt{2} \text{inverf}(1-2\mu_j)$  wherein  $\mu_j$  and  $\sigma_j$  are desired mean and variance, respectively, at the vector index  $j$  and inverf is an inverse error function.

35. The system of claim 32 wherein the program code causing the processor to construct the normal vector causes the processor to:

generate normal elements of the normal vector using the desired mean vector and the first covariance matrix.

36. The system of claim 35 wherein the program code causing the processor to generate the sampling vector causes the processor to:

3 compare a normal element  $Y_k$  of the normal vector at a vector index  $k$  with a  
 4 corresponding threshold element  $\tau_k$  of the threshold vector at the vector index  $k$ ;  
 5 set a sampling element of the sampling vector at the vector index  $k$  to a first value  
 6 if the normal element  $Y_k$  is greater than the corresponding threshold element  $\tau_k$ ; and  
 7 set the sampling element of the sampling vector at the vector index  $k$  to a second  
 8 value if the normal element  $Y_k$  is equal to or less than the corresponding threshold element  
 9  $\tau_k$ .

1 37. The system of claim 32 wherein the program code causing the processor to  
 2 generate the integral expression  $F$  causes the processor to:

3 form a first variable  $\rho = s_{ij}/(\sigma_i \sigma_j)$ ;

4 form a second variable  $c = \sqrt{2(1 - \rho^2)}$ ;

5 form a third variable  $P = (a_i + a_j)/(c\sqrt{2})$ ,  $P$  being one of the integral limits;

6 form a fourth variable  $Q = (a_j - a_i)/(c\sqrt{2})$ ; and

7 form the integral expression

$$8 \quad F(\rho) = \frac{\sqrt{1-\rho}}{2\sqrt{\pi}} \int_P^{\infty} e^{-p^2(1-\rho)} (erf(\sqrt{1+\rho}(Q+p+P)) - erf(\sqrt{1+\rho}(Q-p+P))) dp$$

9 wherein  $p$  is an integral variable,  $erf$  is an error function,  $a_i$  and  $a_j$  are respectively  
 10 equal to  $(\tau_i - \mu_i)/\sigma_i$  and  $(\tau_j - \mu_j)/\sigma_j$ ,  $\tau_i$  and  $\tau_j$  being the threshold elements at the vector indices  
 11 equal respectively to the row index  $i$  and the column index  $j$ ,  $\mu_i$  and  $\mu_j$  being the means at  
 12 the vector indices equal respectively to the row index  $i$  and the column index  $j$ ,  $\sigma_i$  and  $\sigma_j$   
 13 being the variances at the vector indices equal respectively to the row index  $i$  and the  
 14 column index  $j$ .

1 38. The system of claim 32 wherein the program code causing the processor to  
 2 obtain the first non-diagonal element causes the processor to:

3 determine a right hand side (RHS) quantity  $g_{ij} = \Sigma_{ij} + \mu_i \mu_j$ ;

4 equate the integral expression to the RHS quantity to form an integral equation  $F =$   
 5  $g_{ij}$ ; and

6 solve the integral equation for the first variable  $\rho$ .

- 1           39.    The system of claim 38 wherein the program code causing the processor to  
2 solve the integral equation causes the processor to:  
3           solve the integral equation using a numerical method.
- 1           40.    The system of claim 36 wherein the first value is 1 and the second value is  
2 0.

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